J. Pack. Sci. Technol. Vol.5 No.4 (1996)

A Theoretical Model of Redemption System when Consumer Inconvenience can be Described as a Linear Function of Distance

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Waste reduction policy by promotion of recycling has been introduced in several countries. "Redemption System" in the state of California (U.S.A.) has achieved its goal at least cost. This study illustrates the optimal features of this system in achieving the target recycling rate within budget constraints. We take into account for consumer inconvenience as cost and we consider the case when the inconvenience of the consumers to return can be described as a linear function of the distance from the collection stations. The theoretical analysis revealed that the optimum refund size does not depend on the target recycle rate. Thus, a policy of dispersing the collection stations and not raising the refund rate resulted in the optimum performance.

Keywords: Recycling, Redemption system, Post-consumer waste, Inconvenience

1. Introduction

1.1. Background

Municipal waste management is now one of the major policy issues in developed countries. They all are having difficulties constructing new facilities, such as incinerators and land fills, due to environmental protection concerns, and they also face increasing waste management costs.

To reduce waste and avoid land fill, these countries are being obliged to adopt recycling policies. In the USA, ten states have introduced a deposit refund system since the 1980's and many municipalities have adopted a curb-side collection system. In European countries a variety of economic instruments for recycling were introduced in the 1980's. Recycling has become a more political and international issue since Germany and France decided to adopt more strict mandatory deposit systems at the beginning of the 1990's. These may exert some influence on international economic and trade system. Germany's new law, in particular, requires packaging-related industries to recycle and reuse their packaging.

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1.2. Redemption system

Of all the systems so far, a unique deposit refund system called the "Redemption system" in California has achieved its goals at least cost and accomplished this over the shortest time period. This system is unique in several aspects. First, the state originates and handles deposits and retailers pay deposits only to the state. This makes the system more integrated and efficient, since it eliminates the handling of returned containers by retailers or brand holders. Second, the system is cheaper than other deposit-refund systems because the returned containers are redeemed at "recycling centers" which are widely dispersed and close to residential areas. In contrast, other deposit refund systems have higher costs because all retailers must redeem containers on their own premises. Third, this system provides consumers with the choice of returning containers to recycling centers if they want refunds or to curb-side locations if they prefer such convenience.

The performance of this Redemption system provides some very interesting data about consumer preference between the monetary refund (economic incentives) and the inconvenience of returning the beverage container (distance to the collection point).

For higher recycling rates, policy alternatives are limited to providing many convenient recycling centers or increasing the redemption size (value). Both increase total system cost. At the same time, total revenue, determined by the recycling rate and deposit value, must balance total cost. Total cost consists of the capital and operating cost of the system and the total redemption paid for returned containers.

2. Theoretical model

This study is intended to illustrate the optimal features subject to achieving a given recycling rate and within budget constraints.

In the theoretical model, the total system costs plus the inconvenience to consumers are minimized for a given recycling rate. We hypothesized that inconvenience is an increasing function of the distance that consumers have to travel to return containers to a recycling center. The analysis seeks to determine which costs would be less: setting a higher redemption rate or dispersing many recycling centers.

Figure 1 illustrates a process flow of the "Redemption system." Among the variables used, number of stations, refund size, and deposit size are policy variables constrained in that income should balance with expenditure.

In the present model we assume:

1) The target recycling rate is given.

2) The inconvenience of a consumer is a linear function of the distance to the collection center.

3) The operation cost is a sum of the fixed cost, in turn, is proportional to the number

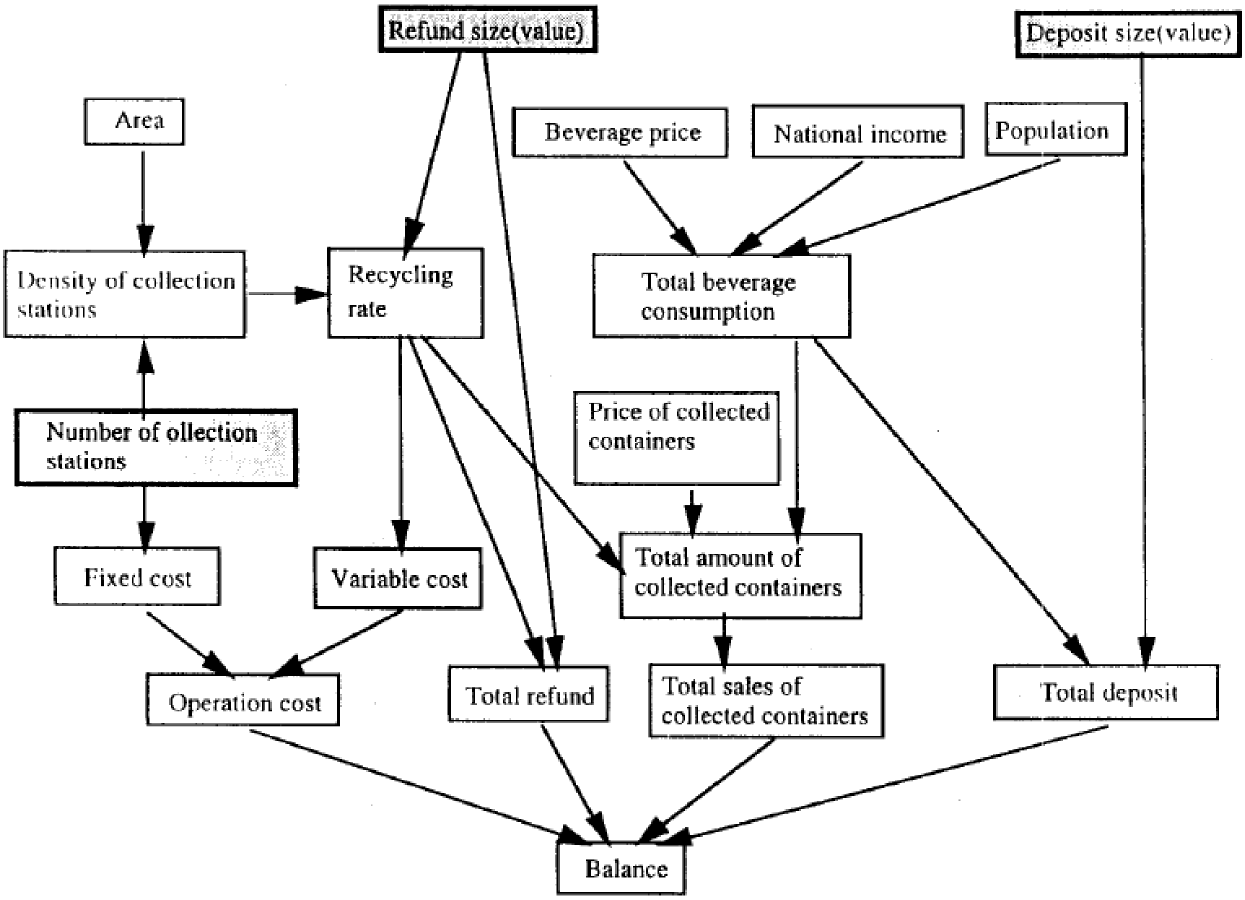


Fig. 1 System flow diagram of the redemption system

of stations and the variable cost. This in turn, is proportional to the amount of the collected containers.

4) The effect of deposit size on the purchasing behavior of consumers is small.

Let us consider the social cost (SC) to be the sum of the decrease of total surplus including the total refund (TS), the inconvenience of consumers (TI) and the externalities (DE), such as decrease of capacity of land fill site. Thus,

$$SC = TS + TI + DE \qquad (1)$$

If we have a social consensus on the quantitative evaluation of DE, the optimal recycling rate would be obtained from the condition of minimization of the social cost. However, the DE consensus is very difficult to reach. Accordingly, the practical target is the minimization of the social cost under a given recycling rate. When we consider the possibility of social consensus, financial balance of the redemption system would of course be critical.

The problem can be described as:

Min

$$SC(r,d,\rho)=TS+TI+DE \quad (2)$$

s.t.

$$R(r,\rho)=R_0 \quad (3)$$

$$OC+TR=TD+RV \quad (4)$$

In the present model the cost for processing of the collected containers is not treated explicitly. This is because processing cost would be reflected in the price of the collected containers.

Given that change in the sale of beverages due to the introduction of the redemption system is small, the decrease of total surplus, including the total refund (TS) can be approximated as (Appendix A[1)]):

$$TS=TD-TR \quad (5)$$

The problem can then be rewritten as:
Min

$$TSC(r,d,\rho)=OC-RV+TI+DE \quad (6)$$

s.t.

$$R(r,\rho)=R_0 \quad (3)$$

$$OC+TR=TD+RV \quad (4)$$

Since the change of total beverage sales is small and the target recycling rate is given as a constant, sales of the recycled containers (RV) and externalities (DE) are nearly constant.

We can then simplify:

Min

$$SC(r,d,\rho)=OC+TI \quad (7)$$

s.t.

$$R(r,\rho)=R_0 \quad (3)$$

$$OC+TR=TR+RV \quad (4)$$

We next assume the following to formulate the inconvenience of consumers:

5) Collection stations are located on the nodes of a normal grid.
6) Consumers would decide to refund or not by comparing the refund price and the inconvenience after the consumption of the beverage.

From assumptions 2 and 6, there should be a critical distance (x_c) at which inconvenience is equal to refund size. Consumers in the area where the distance to the recycling station is less than x_c would choose to refund; the rest would discard the containers (**Figure 2**).

The recycling rate can be expressed as the ratio of the hatched area of **Figure 2**.

A simple geometrical calculation (Appendix B) yields;

$$R=\frac{\pi}{4}\theta\, f_1(\theta) \quad (8)$$

$$\begin{cases} 0<\theta<1 \\ \quad f_1(\theta)=1 \\ 1<\theta<2 \\ \quad f_1(\theta)=1-\frac{4}{\pi}\left(\tan^{-1}\sqrt{\theta-1}-\frac{\sqrt{\theta-1}}{\theta}\right) \end{cases} \quad (9)$$

where

$$\theta = 4\rho x_c^2 \quad (10)$$

The function $f_1(\theta)$ exhibits the correction of the effect of accumulating refund circles shown in **Figure 2**. From assumption 2, the inconvenience of a consumer at the edge of the refund circles ($I(x_c)$) is equal to the refund size:

$$I(x_c) = r \quad (11)$$

Therefore, the critical distance can be described by refund size as:

$$x_c = a_1 r + a_2 \quad (12)$$

The relation between refund size and amount of recycled containers is depicted in Figure 3. In **Figure3**, the sum of S_1 and $S_2(RrQ)$ indicates the total amount which is refunded to consumers. The increasing curve in **Figure 3** can be interpreted as the supply curve of the used containers, thus the area below the supply curve, S_2, is the total consumer inconvenience. R can be expressed as a function of r by using equations (8), (10), and (12). Thus,

$$R(r,\rho) = \pi\rho\left(a_1 r + a_2\right)^2 f_1\left(4\rho\left(a_1 r + a_2\right)^2\right) \quad (13)$$

TI is;

$$TI(r,\rho) = Q\left(R(r,\rho)\, r - \int_0^r R(t,\rho)\, dt\right) \quad (14)$$

According to assumption 3, operation cost (OC) is the sum of the fixed cost (C_s) and the variable cost. The fixed cost is proportional to

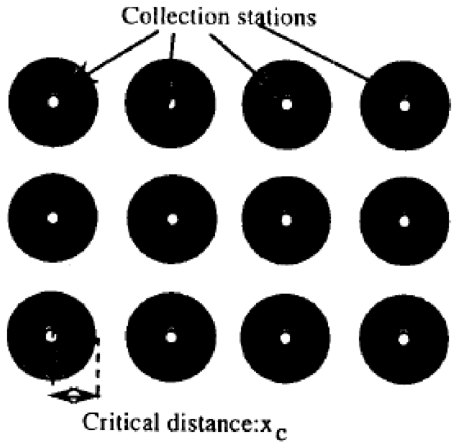


Fig. 2 Collection station and critical distance

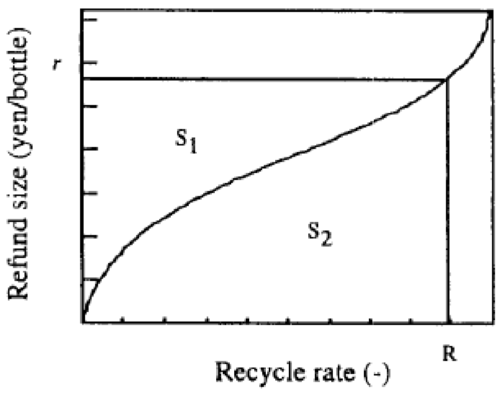


Fig. 3 Recycle rate and refund size

the number of stations and the variable cost (C_v), which, in turn, is proportional to the amount of recycled containers.

$$OC = C_s + R \cdot Q \cdot C_v \quad (15)$$

and

$$\frac{C_s}{Q} = \frac{N \cdot C_f}{Q} = \frac{\rho \cdot C_f}{\rho_p \cdot \gamma} \quad (16)$$

Substitution of equation (16) into (15) yields the following equation,

$$OC(r,\rho) = Q\left(\frac{C_f}{\rho_p \gamma}\rho + C_v R(r,\rho)\right) \tag{17}$$

By substitution of equations (14), (17), (8), and (10) into (7) and (3), we can derive this expression.

Min

$$SC(r,\rho) = Q\left(C_v + r\right)R(r,\rho) + Q\rho \left\{\frac{C_f}{\rho_p \gamma} - \pi\int_0^r \left(a_1 t + a_2\right)^2 f_1\left(4\rho\left(a_1 t + a_2\right)^2\right)dt\right\} \tag{18}$$

s.t.

$$R = R_0 = \pi\rho\left(a_1 r + a_2\right)^2 f_1\left(4\rho\left(a_1 r + a_2\right)^2\right) \tag{19}$$

The solution for r is;

$$r^* = F^{-1}\left(\frac{C_f}{\pi\rho_p\gamma}\right) \tag{20}$$

where

$$F(r) = \int_0^r \left(a_1 t + a_2\right)^2 \left\{ f_1\left(\left(\frac{a_1 t + a_2}{a_1 r + a_2}\right)^2 \theta_0\right) + \left(\frac{a_1 t + a_2}{a_1 r + a_2}\right)^2 \theta_0 \frac{df_1(\theta)}{d\theta}\right\}dt \tag{21}$$

Station density and deposit size are given by,

$$\rho^* = \frac{\theta_0}{4\left(a_1 r^* + a_2\right)^2} \tag{22}$$

$$d^* = R_0\left(C_v + r^* - v\right) + \frac{C_f}{\rho_p \gamma}\rho^* \tag{23}$$

In the case when individual refund circles are set apart,$f_1(\theta)$, is unity from equation (9). Then the optimum solution becomes

$$r^\# = F^{\#-1}\left(\frac{C_f}{\pi\rho_p\gamma}\right) \tag{24}$$

$$F^\#(r) = \int_0^r \left(a_1 t + a_2\right)^2 dt \tag{25}$$

$$\rho^\# = \frac{R_0}{\pi\left(a_1 r^\# + a_2\right)^2} \tag{26}$$

From equations (24) and (25), it is evident that the optimum refund size is independent from the target recycling rate. It only depends on consumer behavior (I(x), γ)> fixed cost of collection station (C_f), and population density (ρ). Station density is proportional to the target recycling rate (Equation 26).

Let us consider the most simple case when a_2 is null and the target recycle rate is less than $\pi/4$. The optimum refund size, the optimum station density are:

$$r^{\#\#} = \left(\frac{3C_f}{\pi\rho_p\gamma a_1^2}\right)^{\frac{1}{3}} \tag{27}$$

$$\rho^{\#\#} = \left(\frac{1}{\pi}\right)^{\frac{1}{3}}\left(\frac{\rho_p\gamma}{3C_f a_1}\right)^{\frac{2}{3}} R_0 \tag{28}$$

It is remarkable that the optimum refund size is insensitive to the fixed cost of collection station (C_s). On the other hand, consumer behavior (a_1) affect much on the optimum refund size.

The social cost is:

$$\begin{aligned} SC^{\#\#} &= Q\left(C_v + r^{\#\#}\right)R_0 \\ &= Q\left\{C_v + \left(\frac{3C_f}{\pi\rho_p\gamma a_1^2}\right)^{\frac{1}{3}}\right\}R_0 \end{aligned} \tag{29}$$

The social cost described by equation (29) does not include RV and DE. The total social cost is:

$$TSC^{**} = Q\left(C_v + r^{**}\right)R_0 - RV + DE$$
$$= Q\left\{C_v + \left(\frac{3\,C_f}{\pi\,\rho_p\,\gamma\,a_1^2}\right)^{\frac{1}{3}} - v\right\}R_0 + DE(R_0) \quad (30)$$

In equation (30), quantitative description of the externality term $DE(R_0)$ is very difficult. However, from this equation, we can obtain the necessary condition for the rationalization of the enhancement of recycling within budget constraints as:

$$\frac{dDE(R_0)}{Q\,dR_0} \leq v - C_v - \left(\frac{3\,C_f}{\pi\,\rho_p\,\gamma\,a_1^2}\right)^{\frac{1}{3}} \quad \left(R_0 \leq \frac{\pi}{4}\right) \quad (31)$$

The left-hand side of equation (31) is a marginal decrease of externalities per a recycled container. The first and the second term of the right-hand side are the marginal income per a recycled container, the third term of the right-hand side implies the consumer inconvenience per a recycled container.

3. Conclusion

Theoretical analysis proved that when we minimize the sum of operation cost of a redemption system and inconvenience of consumers under given target recycling rate, the optimal refund size is not depend on the target recycling rate up to $\pi/4$.

The optimum refund size is insensitive with the fixed cost of collection stations.

Acknowledgment

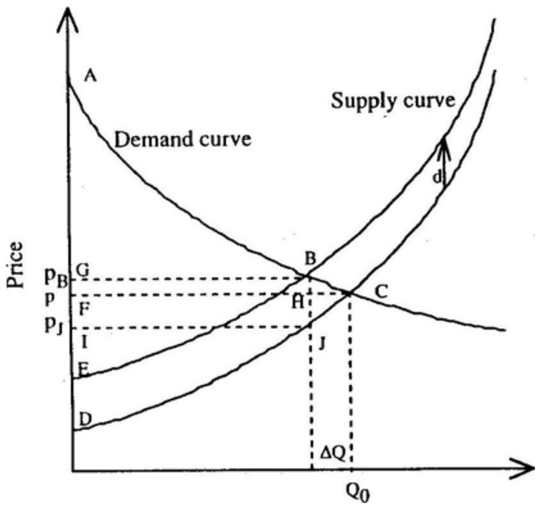
The author would like to appreciate Professor Yoshifusa Kitabatake of Graduate School of Kyoto University, Professor Akinori Kumagaya of Aoyama Gakuin University, and Dr. Hiroshi Asano and Dr. Isamu Matsukawa of CRIEPI, Professor Tsutomu Tanaka of Chuo University, Mr. Isoroku Sawada and Mr. Masaru Eto of Economic Planning Agency, Mr. Hiroshi Tsubouchi of National Land Agency for valuable comment.

This study was done as a part ofa project of Economic Research Institute in which the author collaborated with Professor Yoshifumi Fujii of Bunkyo University, Professor Kazuhiro Ueta of Kyoto University, Professor Eiji Hosoda of Keio University, Miss Mihoko Shimamoto of Hosei University, Mr. Kazuhiro Noma, Mr. Toshiyuki Suzuki, and Mr. Hiroyoshi Nakayama of Economic Planning Agency: the author appreciate much for sincere discussion and helpful comments.

Appendices

Appendix A Total surplus

A supply curve and a demand curve are depicted in **Figure A-1**. Since the price of beverages increase by deposit size d after the introduction of redemption system, the supply curve (DJC) would change vertically (EB).



Quantity
Fig. A-1 Change of total surplus by introduction of redemption system

The equilibrium point shift from C to B, thus the total surplus not considering the refund change from the area ACDA to the area ABEA. The supply elasticity (η_s) and demand elasticity (η_d) are defined as:

$$\begin{cases} \eta_d = -\dfrac{p_o}{Q_0}\dfrac{\Delta Q}{p_o - p_B} \\ \eta_s = \dfrac{p_o}{Q_0}\dfrac{\Delta Q}{p_o - p_J} \end{cases} \qquad \text{(A-1)}$$

and

$$P_B - P_J = d \qquad \text{(A-2)}$$

We obtain pB and pJ as:

$$\begin{cases} p_B = p_o + \dfrac{\eta_s}{\eta_d + \eta_s} d \\ p_J = p_o - \dfrac{\eta_d}{\eta_d + \eta_s} d \end{cases} \qquad \text{(A-3)}$$

Thus the decrease of total surplus TS' (GBCJIG) is:

$$TS' = Q_0 d \left(1 - \frac{1}{2} \frac{\eta_d \eta_s}{\eta_d + \eta_s} \frac{d}{p_o} \right) \qquad \text{(A-4)}$$

In equation (A-4), because η_s and η_d are usually less than unity, and d is much smaller than p_o, the second term is much smaller than the first term. According to the study of Porter on the cost benefit analysis of deposit-refund system, the second term of equation (A-4) is less than 2% of the first term[2)]. Therefore, the change of total surplus including refund (TS) is:

$$\begin{aligned} TS &= Q_0 \{ d - Rr \} \qquad \text{(A-5)} \\ &= TD - TR \end{aligned}$$

Appendix B Recycling rate and critical distance

When collection stations are located in the nodes of normal grids of size L, consumers within the critical distance from collection stations would return their containers(**Figure B-1**). The recycling rate is the ratio of area of the refund circles to the total area.

The recycling rate (R) is:

$$\begin{cases} x_c < \dfrac{L}{2} \\ \quad R = \pi \cdot \left(\dfrac{x_c}{L}\right)^2 \\ x_c > \dfrac{L}{2} \\ \quad R = \pi \cdot \left(\dfrac{x_c}{L}\right)^2 - \left(\dfrac{2x_c}{L}\right)^2 \cdot \tan^{-1}\sqrt{\left(\dfrac{2x_c}{L}\right)^2 - 1} \\ \qquad + \sqrt{\left(\dfrac{2x_c}{L}\right)^2 - 1} \end{cases} \qquad \text{(B-1)}$$

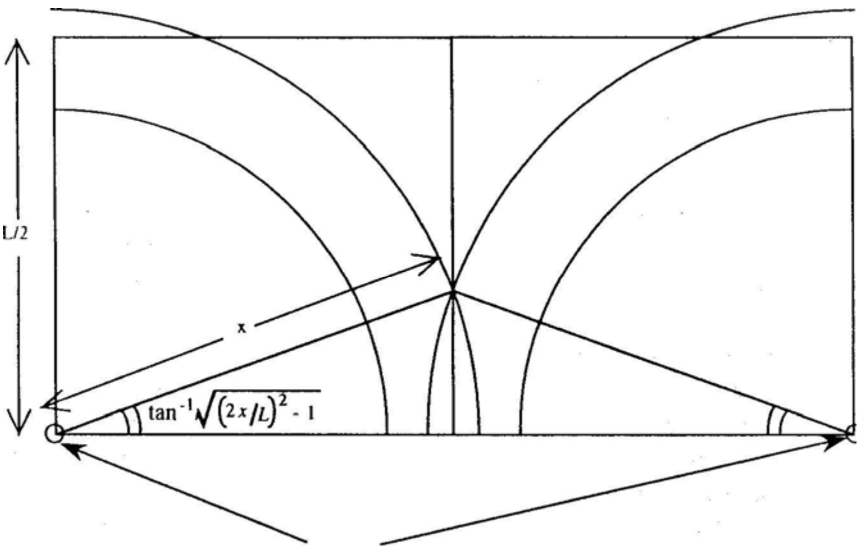


Fig. B-1 Collection station and critical distance

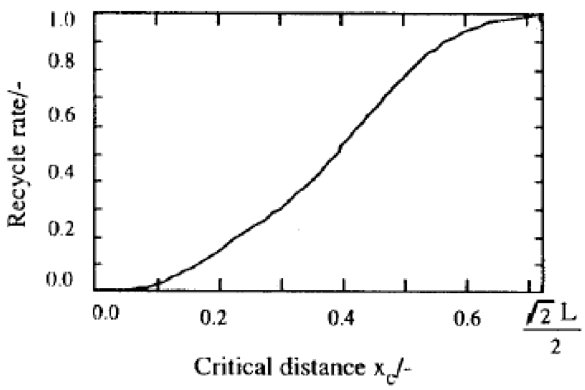


Fig. B-2 Recycle rate and critical distance

Using station density of collection stations (ρ), equation (B-l) can be rewritten as:

$$\begin{cases} x_c < \frac{L}{2} = \frac{1}{2\sqrt{\rho}} \\ \quad R = \pi \rho x_c^2 \\ \frac{1}{2\sqrt{\rho}} = \frac{L}{2} < x_c < \frac{L}{\sqrt{2}} = \frac{1}{\sqrt{2\rho}} \\ \quad R = \pi \rho x_c^2 - 4 \rho x_c^2 \tan^{-1} \sqrt{4 \rho x_c^2 - 1} \\ \qquad + \sqrt{4 \rho x_c^2 - 1} \end{cases} \tag{B-2}$$

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Nomenclature

a_1	km container/yen	parameter
a_2	m	critical distance for return without refund
C_f	yen/station/year	Cs/N
C_s	yen/year	fixed cost which is proportional to the number of stations
C_v	yen/container	variable cost which is proportional to a recycled bottle
D	km	distance from home to a collection station
d	yen/container	deposit size
DE	yen/year	externalities
DS	yen/container	deposit or refund size
F(r)	yen km^2	function
$f_1(\theta)$	-	function
I(x)	yen/container	inconvenience of consumer described as a function of distance from recycling station
N	-	number of stations
OC	yen/year	operation cost
p	yen/beverage	price of beverage
p_0	yen/beverage	price of beverage before the introduction of redemption system
Q	containers/year	total sales of beverages
Q_0	containers/year	total sales of beverages before the introduction of redemption system
R	-	recycling rate
r	yen/container	refund size
R_o	-	target recycling rate
RV	yen/year	sales of the recycled bottles
SC	yen/year	social cost
TD	yen/year	total deposit
TI	yen/year	total inconvenience of consumers
TR	yen/year	total refund
TS	yen/year	change of the total surplus including refund
TSC	yen/year	total social cost including the sales of recycled materials and externalities
v	yen/container	price of recycled material
x_c	km	critical distance where inconvenience is equal to refund size
η_d	-	demand elasticity
η_s	-	supply elasticity
γ	containers/year	beverage consumption unit
θ	-	parameter
ρ	$1/km^2$	density of collection stations
ρ_p	$1/km^2$	population density

Superscript

*	optimum
#	target recycle rate is less than $\pi/4$
##	target recycle rate is less than $\pi/4$ and a_2 is null

(Received 4 June 1996)
(Accepted 7 November 1996)